Maxwell's distribution law of molecular speeds (c) →

Used Boltzmann's distribution law.

$dN(c)\,dc \Rightarrow$ No. of molecule in dc range

dc

$c \qquad c+dc$

$dN(c)\,dc$

$p \qquad p+dp$

$\epsilon \qquad \epsilon+d\epsilon$

$K \qquad K+dK$

If gas contain N molecules → momentum

$$\frac{dN}{N} = dP = \frac{g(p)\,dp\,e^{-\beta\epsilon}}{Z}$$

dP ↓ Probab. ; $g(p)$ → No. of microstate or degeneracy

$$= A\,g(p)\,dp\,e^{-\beta\epsilon}$$

$$= A\,\frac{V4\pi p^2}{h^3}\,dp\,e^{-\beta\epsilon}$$

$$dP = \frac{AVp^2}{2\pi^2\hbar^3}\,e^{-\beta(p^2/2m)}\,dp$$

$$\int dP = \frac{AV}{2\pi^2\hbar^3}\int_0^\infty e^{-\beta(p^2/2m)}\,p^2\,dp$$

$$1 = \frac{AV}{2\pi^2\hbar^3}\sqrt{\frac{2\pi}{(\beta/4m)^3}}$$

$$\boxed{A = \frac{(2\pi)^{3/2}\hbar^3}{V}\left(\frac{\beta}{4m}\right)^{3/2}}$$

$$dP = \frac{4}{\sqrt{\pi}}\left(\frac{2}{2mK_BT}\right)^{3/2} p^2\, e^{-\beta p^2/2m}\, dp$$

$p = mc$

$$\boxed{dP = \frac{dN}{N} = 4\pi\left(\frac{m}{2\pi K_BT}\right)^{3/2} e^{-mc^2/2K_BT}\, c^2 dc}$$

$c \to 0$ to ∞

$$\boxed{dN = 4\pi N\left(\frac{m}{2\pi K_BT}\right)^{3/2} e^{-mc^2/2K_BT}\, c^2 dc}$$

<u>Average speed</u>: $(\bar{c})$

$$\bar{c} = \frac{\int c\, dP}{\int dP}$$

$$\int dP = 1$$

$$\bar{c} = \int c\, dP$$

$$= \int_0^\infty c\,\frac{dN}{N} = \frac{\int c\, dN}{\int dN}$$

$$\boxed{\underset{\text{Average speed}}{\bar{c}} = \sqrt{\frac{8K_BT}{\pi m}}}$$

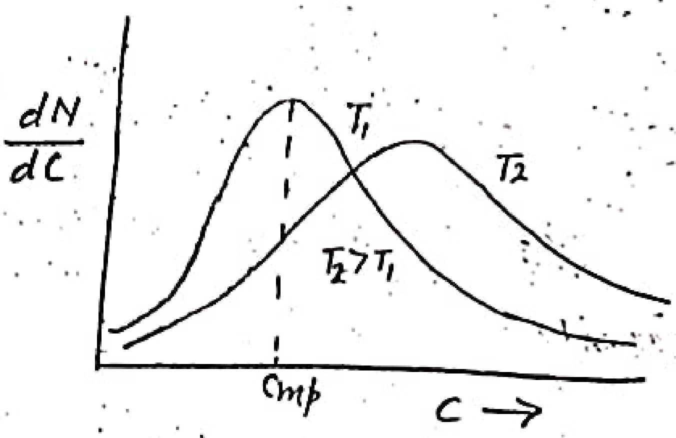
$$\overline{c^2} = \frac{\int c^2 dN}{\int dN}$$

$$= \frac{3K_BT}{m}$$

$$\boxed{\left(\overline{c^2}\right)^{1/2} = \sqrt{\frac{3K_BT}{m}} = C_{rms}\ \text{(root mean square)}}$$

$$\underset{\text{root mean square}}{C_{rms}} > \underset{\text{Ave}}{\bar{C}} > C_{mp}\ \text{(most probable)}$$

$$\sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$$



$A_1 = A_2$ (area)

$\int \frac{dN}{dc} \cdot dc = N$

Breadth × height

Most probable speed (C_{mp}):

followed by max. no. of molecules.

at $C = C_{mp}$

$$\frac{d}{dc}\left(\frac{dN}{dc}\right) = 0$$

$$\rightarrow \frac{dN}{dc} = 4\pi N \left(\frac{m}{2\pi K_B T}\right)^{3/2} c^2 e^{-mc^2/2K_BT}$$

peak depend upon temp.